Encouraging the defragmentation of habitat across privately-owned lands

Pierce Donovan[†]

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Abstract

Endangered species conservation is a public issue that often resides on private lands. As land is developed, habitat fragmentation weakens natural ecosystem function and decreases the probability of long-run species viability. Conservation banking is a modern solution to habitat loss which can additionally mitigate habitat fragmentation when there is potential for a large protectorate under the control of a single landowner. Unfortunately, land ownership itself is often fragmented, and it is in precisely this scenario where habitat fragmentation is most likely! To address this issue, this paper adapts the banking approach to regulate a decentralized group of landowners. The solution is then applied to a model of neighboring, independent landowners. The aggregation and clustering of conserved lands is manipulated via land use policy, and the value of the resulting landscape is determined.

Keywords: land use, habitat fragmentation, decentralized conservation, conservation incentives, policy design, statistical mechanics, Ising model (JEL codes: C6, Q2, Q5)

[†]Department of Economics, Colgate University
email: pdonovan@colgate.edu **web**: piercedonovan.github.io

This is a work in progress and I'd appreciate any feedback you may have for me. Please reach out with any questions or comments.

1 A public conservation problem on private lands

In the absence of compensation for ecosystem services or species preservation, the social values of land conservation will be ignored by the typical landowner. Holding habitat as an asset won't earn them a return on investment. They will naturally choose to employ their land in another use if there is no way for them to capitalize the social benefits accrued to others. Without any payment for the services that habitat provides society, it is inevitable that most land use decisions will erode these social values (Mandle et al., 2019).

The habitat conservation problem is potentially sadder than this—the social benefits of conserved habitat increase with the strength of the linkages between neighboring parcels, not just the number of parcels conserved (Fox and Nino-Murcia, 2005). Thus there are two positive externalities being ignored as land is developed and habitat is destroyed: (1) the social benefits of conserving any one parcel, and (2) the positive interactions between better-connected parcels and larger clusters of conserved areas.

Land ownership itself is often fragmented in areas of conservation interest (Mandle et al., 2019). Even in cases where the resource benefit is captured by the landowners—e.g. in the case of oil reserves—individuals won't be fully-incentivized to make improvements to their land when returns are shared by the group (for a recent example, see Leonard and Parker (2021)). Collective ownership (of, in our case, the social, ecological benefit) will naturally get in the way of conserving a large, contiguous piece of land—the preference of environmentalists and ecologists; it will naturally inhibit maximal land values as well.

The habitat conservation scenario is one full step behind the collective oil extraction problem because of the lack of private return for habitat. In order for land use policy to properly incentivize some socially-optimal amount of conservation, it must (1) provide a compensating mechanism for social value capture and (2) reflect the positive network effects that the clustering of conserved parcels create. This paper provides a model framework for implementing that policy. Before I outline my solution, I will first describe a prominent natural capital-based approach and its shortfalls related to these two goals.

An existing habitat preservation solution: conservation banking

"Banking" solutions—in which landowners are compensated by nearby land developers to establish conservation easements on their property—have protected large swaths of privately-owned land in the United States.[1] A developer's willingness-to-pay for con-

[1] This introduction merges discussion on both conservation banking and wetlands mitigation banking. The difference between the two concepts is that the former preserves existing habitat while the latter aims to restore new habitat (Mandle et al., 2019). The shortfalls discussed below apply to both banking solutions.

servation arises from (1) a legal requirement to mitigate any unavoidable environmental impacts related to their activities (e.g. reducing, modifying, or degrading habitat, or isolating species) and (2) an option to compensate someone else (the banker/creditor) to mitigate these damages offsite (Ruhl et al., 2005).

The value of this regulatory-driven mitigation strategy is derived from the flexibility in how that mitigation may be achieved. Mitigation actions to be completed on-site are excessively costly to the developer (who has little expertise in the practice), and isolated pockets of habitat provide little environmental benefit (due to lower carrying/dispersal capacity) (van Teeffelen et al., 2014). Allowing mitigation to be performed offsite is a potential boon for both developers (who can lower their cost of compliance) and species of conservation concern (through larger, better-connected and higher-quality conservation areas) (Fox and Nino-Murcia, 2005; van Teeffelen et al., 2014).

Once a banker has committed to setting up a conservation easement, they are entitled to sell a certain number of development credits determined by ecological assessment.[2] After a similar assessment of the damages from a potential development project, a developer may mitigate their environmental impact offsite by purchasing these credits. In general, these credits have been lucrative (Fox and Nino-Murcia, 2005). Since it is cheaper to restore and maintain a larger contiguous parcel rather than a collection of fragmented parcels with the same area, the policy is a boon for bankers as well.[3]

Conservation banking provides an opportunity to "transform a former legal liability (i.e. the species) into a financial asset (i.e. the credit)" (Fox and Nino-Murcia, 2005). This aligns landowner incentives with those of environmentalists, enables developers to settle their mitigation requirements cheaply and quickly, and secures [potentially] superior habitat and ecosystem services (Ruhl et al., 2005).[4]

I will now discuss four primary concerns with the conservation banking approach that limit its real-world use. Taking the vanilla conservation banking approach to be our benchmark policy, I will explain how we can adapt banking policy to ensure (1) the distribution of conserved lands is favorable, (2) the new policy motivates private sector engagement at all scales, (3) the reward scheme for conservationists (and thus the resulting landscape) reflects social values, and (4) the appraisal process is made more efficient.

[2]The conservation easement must be maintained in perpetuity. To support this promise, an endowment fund from the proceeds of credit sales is withheld for monitoring and enforcement (Ruhl and Salzman, 2006).

[3]Credits are bought and sold within a limited geographic range called a service area. This limits potential market activity to ensure that conserved and developed lands are nearby and have similar ecological characteristics. As development within a service area continues, existing credits become more scarce—and valuable. This increases the rents to establishing new conservation easements and spurs bank creation.

[4]Outside of the U.S., the conservation banking strategy is more commonly described by the term *biodiversity offset* (Bull et al., 2013; Gardner et al., 2013; McKenney and Kiesecker, 2010).

Concern 1: Which lands get conserved?

Conservation banking, like any cost-effective strategy, will conserve the land that is least desirable for development. This necessitates a shift of habitat from urban to rural areas, changing the spatial distribution of conservation benefits—and the group of people who receive them (Ruhl and Salzman, 2006). Further, it is inevitable that fewer people will benefit from redistribution of habitat as it moves farther from urban centers (Mandle et al., 2019). Even if it is argued that the value of habitat conserved is greater than what was lost—those that have lost may not be so thrilled.

In many cases, there is a negative correlation between the desirability of a parcel (for development) and the value that parcel has as habitat. Here, there is no economic trade-off being made at the parcel level—each parcel has an obvious use, because local scarcity is not being rewarded. In a sense, the market for credits is incomplete, as the resulting distribution of habitat is not accounted for in the price of a credit. "Urban credits" would theoretically be worth more than rural ones, all else equal, if this value was reflected.

Concern 2: Who is incentivized to conserve land?

Conservation banking does not motivate private sector engagement at all scales (Mandle et al., 2019); the average size of new banks is approaching one thousand acres, a far cry from typical land holdings (Fox and Nino-Murcia, 2005; Carreras Gamarra and Toombs, 2017). Smallholders do not have much to gain from establishing a conservation easement, since the social value of a small and isolated pocked of wetlands or species habitat is fairly small. The tragedy, of course, is that many small parcels conserved as a group are worth significantly more than the sum of the values of each individual piece conserved in isolation. The lack of regard for this spatial externality leads to the continued losses of fragmented and isolated habitat, rather than the clustering of these parcels.

There is clearly a need to make an impression on smallholders in order to coordinate their actions over space. The most explicit way to do this is to increase the credit reward to landowners who conserve as their neighbors align their land-use choices similarly. Compensation that rises in the number of actively conserved neighboring parcels accurately reflects the positive spatial externality and incentivizes both early- and late-movers to dedicate their land to a conservation easement.

Concern 3: What are credits awarded for?

Bankers are speculators of future local development; without sufficient credit market activity, no new lands will be conserved (Ruhl et al., 2005; van Teeffelen et al., 2014). Less-desirable "like-for-similar" trades between ecosystem types may occasionally be needed to improve credit market health, but this brings into question what the credits represent.

It is difficult to equate one patch of habitat with another (Ruhl et al., 2005). Credits are ideally awarded for specific impacts, like the ability of a wetland to filter or impede water flowing through it, or the number of breeding pairs of an endangered species that a parcel of land supports (Mandle et al., 2019). This does occur in some cases, but the vast majority of projects are valued simply by acreage conserved, a proxy that muddies the comparison of debtor and creditor land (Ruhl and Salzman, 2006; Carreras Gamarra and Toombs, 2017; Mandle et al., 2019). At small scales, this is fairly problematic, since there is naturally more variability in parcel quality among smaller aggregations being conserved, and thus less of a correlation between size and value (Robertson, 2004; Boisvert, 2015).

If many individuals pool their land to form a cluster of conserved parcels, area may be an acceptable proxy for habitat value as the pool "averages" enough parcels of high and low quality, and the connections between them may elevate the values of each individual piece to sufficient levels. Several potentially cluster-able parcels can be valued as a group, and as individuals conserve their contribution to the cluster, they receive a piece of the total value according its size and the number of neighbors that have similarly contributed.

Concern 4: Bureaucracy and the speed of appraisals

Reducing bureaucratic challenges can further increase the amount of private property voluntarily committed to banking (Fox and Nino-Murcia, 2005; Bunn et al., 2013). In particular, the parcel-by-parcel process for evaluating habitat value is very costly and time-intensive, often taking between two to seven years (Mandle et al., 2019). Additionally, the number of credits required to mitigate a particular development project and the number of credits awarded to a nearby bank are partially determined by a bargaining process based on personal relationships, the value or urgency of a project, and the ability to pay for mitigation, rather than the value of habitat loss or gain (Robertson, 2004; Boisvert, 2015). A better system would separate the assessment from the landowner.

To address these concerns, the U.S. Fish and Wildlife Service (FWS) has proposed the Endangered Species Act Compensatory Mitigation Policy, which suggests a shift from site-based conservation (e.g. focusing on special areas of particular interest) to a more holistic approach that reconciles competing objectives of conservation and development across a larger "landscape" (network of parcels) (U.S. Fish and Wildlife Service, 2016, 2018).[5]

[5]The proposal outlines a new policy objective of no net loss of species viability (e.g. quality/carrying capacity of habitat, number of individuals, size/distribution of population). Wetlands mitigation banking similarly reflects the policy goal of "no net loss" of wetland function in the Clean Water Act. There is no

One way to achieve the desired increases in "consistency, predictability, transparency, and efficiency in implementation of the ESA" (U.S. Fish and Wildlife Service, 2016, 2018) would be the group-level assessment suggested in the section above. This process not only provides more realistic measures of value, but can be done at a much more rapid pace with less of an opportunity for corruption. This collective valuation technique could potentially solve the dual problems of not being able to assess the counterfactual (Carreras Gamarra and Toombs, 2017) or additionality (Sonter et al., 2019) of a a particular easement's creation, since all parcels will be evaluated, regardless of landlord intent.

Conservation policy can address fragmented ownership

The above concerns with conservation banking policy can be realistically addressed. This paper so far has provided some suggestions for changing the compensation mechanism in order to combat habitat fragmentation in the scenario where it is most likely—when land ownership is also fragmented.

Section 2 develops a model of the expected social value (the values derived from both development and conservation of lands) of a network of many neighboring parcels, given the land use decisions of many individual landowners. Value is derived from (1) the number of developed parcels, (2) the number of conserved parcels, (3) the clustering of similar parcel types among neighbors, and (4) the local scarcity of parcels of a certain type. Given these determinants of value, a regulator may tweak the rewards to conservation banking in order to change land use decisions by individual landowners.

Section 3 further discusses this contribution to conservation policy, the next steps for this paper, and more broadly, the applicability of the following modeling framework to many other fields in economics.

2 A model for uniting fragmented lands

The point of this section is to understand how a landscape of interconnected landowners will respond to a land use policy that rewards conservation in the style detailed in Section 1. Section 2.1 develops a simple valuation of the landscape given each landowner's land use decisions. Section 2.2 generalizes this value to its probability distribution, as the land use decision is complex and only partially-determined by some external policy, and Section 2.3 visualizes the expected land use decision given this probability distribution.

analog permitted under the scope of the Endangered Species Act according to the 2017-2020 administration; conservation banking is currently seen as a solution for increasing the area under protection, but not a means of ensuring no net loss of species, habitat, or value (U.S. Fish and Wildlife Service, 2016, 2018).

Finally, Section 2.4 provides the necessary model extensions that fully capture the nuances of the social value defined above.

2.1 The social benefits of uniting fragmented lands

This section establishes the rules for determining V_i, the total value of the landscape of networked parcels in a given configuration i. A configuration is simply a list of all of the land use decisions for each landowner. Landowners may choose to set aside their parcel for conservation or employ it in some competing use that destroys its value as habitat.

We can start with a simple model that captures the desire to reward both the number of conserved parcels as well as the clustering of conserved lands, e.g.,

$$V_i = w \sum_{<j,k>} s_j s_k + u \sum_j s_j, \tag{1}$$

where s_j and s_k identify the uses of parcels (sites of similar area) j and k, and the first sum is over pairs of neighboring parcels. Within each configuration i, there are N parcels, with n set aside for conservation $(s_j = 1)$ and $(1 - n)$ employed in other uses $(s_j = -1)$. Clearly, if $u, w > 0$, V_i is increasing in both conserved parcels and the number of "links" between neighboring conserved parcels.

The parameters u and w provide us with the relative contributions to the social benefit from each factor (area and clustering of conserved lands, respectively).[6] These can be increased or decreased in order to change the reward to those who conserve their lands. Changing the sign of w and u causes the system to favor a very fragmented landscape with lots of economic development. Interestingly, the social value also increases if neighboring parcels match in development status, offsetting the direct negative effect of development. Thus the social value function derives value from both actions. If $u = 0$, then the regulator is signalling no preference over which land use is chosen, and will only care about the clustering effect.

There must be some uncertainty in the realized value resulting from the incentives u and w, since each of the land use decisions are themselves random variables given these stimuli. Thus no configuration i is necessarily guaranteed as we tweak those values. Section 2.2 lays out the underlying principles that govern the variability in individual behavior, and this will allow us to develop the expected value of the landscape given choices for u and w in Section 2.3.

[6]Section 2.4 will introduce additional parameters that will capture the other desirable values beyond conservation and clustering that were listed in Section 1. It also includes guidance for allowing the model to incorporate site characteristics and multi-dimensional land use decisions.

2.2 The probability distribution of social benefits

A large network of many small landowners admits many different possible land use configurations (labeled i) of the parcels, each associated with some value V_i. The expected social benefit $E[V] = \Omega$ will rely on the distribution over the potential configurations that can occur as a result of a particular policy (i.e. particular values for u, w).

Given a particular policy, there will be random deviations in the behavior of each landowner due to other unobservable factors, so the actual aggregated benefits will depart from the expectation. This leads us to ask: what is the appropriate shape of the distribution for the social benefit V, conditional on having expected net benefit Ω? We can rephrase this question: if we were to run the same policy experiment on many identical parcel networks, what is the most likely benefit distribution with mean Ω?

This is a well-established question with a well-known solution (Boltzmann, 1868; Gibbs, 1902). The basic structure bears repeating. We could run our hypothetical experiment N times, and for trial t record the resulting benefits V_t. We then count the number of trials where $V_t = V_i$, i.e. n_i, the number of trials ending in potential configuration i. The most likely V-distribution is the one with counts $\{n_i\}$ that yield the greatest number of combinations of N trials with mean Ω:

$$\max_{\{n_i\}} \binom{N}{n_1, n_2, n_3 \ldots} = \frac{N!}{\prod_i n_i!} \text{ s.t. } \sum_i n_i = N \text{ and } \sum_i n_i \cdot V_i = N \cdot \Omega. \tag{2}$$

If there are many parcels in our network, there will be a great deal of levels $\{V_i\}$, and this problem becomes somewhat untenable. A common next step is to maximize the *log* of the multinomial coefficient above and make use of Stirling's approximation—which assumes that we make the number of trials N large enough that each of the $\{n_i\}$ are large as well. This new maximand looks like

$$S = \sum_{\text{supp}\{p_i\}} -p_i \log p_i, \tag{3}$$

after dropping a factor of N (which does not affect the maximization problem) and defining $p_i = n_i/N$, i.e. the proportion of N trials in configuration i. The sum is over all permissible configurations i that make up the support of $\{p_i\}$ conditional on mean Ω.[7]

The object S is commonly referred to as the *entropy* of the system (Clausius, 1867; Boltzmann, 1868; Gibbs, 1902; Shannon, 1948). Functionally, it is a simple measure that scales with the multinomial coefficient above (it is a concave and monotonic transformation of it);

[7] I will drop the support notation moving forward, since $\lim_{p \to 0^+} p \log p = 0$.

alternatively, it is a measure of the uncertainty represented by the probability distribution $\{p_i\}$ and the thickness of its tails.[8]

We can now choose to find the probability distribution $\{p_i\}$ that maximizes the entropy given that the sum of all the probabilities is 1 and the expected social benefit is Ω. This problem is equivalent to Equation 2 above. Importantly, the principle of maximum entropy allows us to move forward with the derivation of our social benefit distribution without the need to make any unconscious or arbitrary assumptions about each individual's land use decision (for a general argument of the principle, see Jaynes (1957)).

Rewriting the optimization problem in Equation 2 (in the style of Lagrange's method of undetermined multipliers):

$$\max_{\{p_i\}} -\sum_i p_i \log p_i + \alpha \cdot \left(\sum_i p_i - 1\right) + \beta \cdot \left(\sum_i p_i \cdot V_i - \Omega\right), \tag{4}$$

where α and β are the Lagrange multipliers to be determined along with $\{p_i\}$, and the sums are over all permissible configurations i supported by mean Ω (see above).

I will now "invert" the constrained maximization problem (Equation 4), as I would like Ω to be an output, rather than a parameter. This is not as absurd as it is subtle. The above constrained maximization problem simply defines the shape of the distribution for V, and while the mean is the usual candidate for parameterizing a distribution, I would like an alternative parameterization which will free Ω to be a calculable quantity. Since the steps to take are not very obvious, I will show them here.

The first-order condition for the maximum of Equation 4 is

$$\log p_i + 1 - \alpha - \beta V_i = 0 \ \forall i, \tag{5}$$

which we then re-write to provide a formula for p_i, provided we knew the values of our Lagrange multipliers,

$$p_i = \frac{1}{Z} e^{\beta V_i}, \tag{6}$$

where Z is defined as $e^{1-\alpha}$ and assumes the role of the first multiplier. This relationship between p_i and the exponential factor containing V_i is often referred to as the Boltzmann

[8]Some writers may insist on a deeper understanding of entropy via physical example. I am one of those people. Entropy is often described as "disorder," but this is neither an accurate nor helpful definition. Consider a messy bedroom. If it is *your* bedroom, then you likely know where everything is, and if you had misplaced your phone, there would only be a few permissible locations for it to be hiding. But if your friend was to look for your phone, the number of potential locations considered is much higher. Entropy is both a property of the system you are studying and the state of knowledge of the system.

distribution or the Gibbs measure (Boltzmann, 1868; Gibbs, 1902). We can use the law of total probability (our first constraint) to write Z as a function of the social benefit levels V_i and multiplier β,

$$Z = \sum_i e^{\beta V_i}. \tag{7}$$

Z has some truly spectacular properties. To begin the revelations, we use our second constraint to show the relationship between β and Ω:

$$\begin{aligned} \Omega &= \frac{1}{Z}\sum_i e^{\beta V_i} \cdot V_i = \frac{1}{Z}\sum_i \frac{\partial e^{\beta V_i}}{\partial \beta} \\ &= \frac{\partial \log Z}{\partial \beta}. \end{aligned} \tag{8}$$

Lagrange's method does something much more powerful than what it is typically given credit for. Traditionally, we would use our knowledge of $\{V_i\}$ and a desired mean Ω to inform $\{p_i\}$, Z, and β (where Z has assumed the role of α). But since Ω, $\{p_i\}$, Z, and β are interconnected, once one is known, they all are. Put another way, the solution to a constrained maximization problem is the functional relationship between variables. We can invert the problem, if we wish, to be a question of determining Ω given some choice of β and the relationship between them. And all we have done is "flipped the axes" on a graph, allowing β to play the role of independent variable–and Ω the dependent one.[9]

So either β or Ω can fully-parameterize the $\{p_i\}$ distribution, and we have a function that maps information about our network into its expected value, i.e. $\Omega = f(\{V_i\}|\beta)$.[10] This exercise requires another step in order to be truly satisfied with this arrangement. We should feel responsible for creating some "meaning" for β.

What *is* β? Figure 1 presents an illustration of the probability distribution over V for different levels of β. Clearly, as β increases, the probability distribution (Equation 6) becomes more concentrated about the highest potential social benefit values. Conversely, a lower β extends the tail into lower-benefit configurations. So as β increases, Ω—the expected returns to our conservation policy—increases as well.

The social value of the parcel network seems to degrade for low β. If value is partly

[9]Equation 8 will prove to be very valuable—it links this section with the last one. We can also adapt it to determine the expectation of *any* random variable in our model. For example, if $V_i = V_i^0 + \lambda A_i$, then

$$E[A] = \frac{1}{\beta}\frac{\partial \log Z}{\partial \lambda}.$$

[10]Since Z is completely determined by β and $\{V_i\}$, it is omitted from this function.

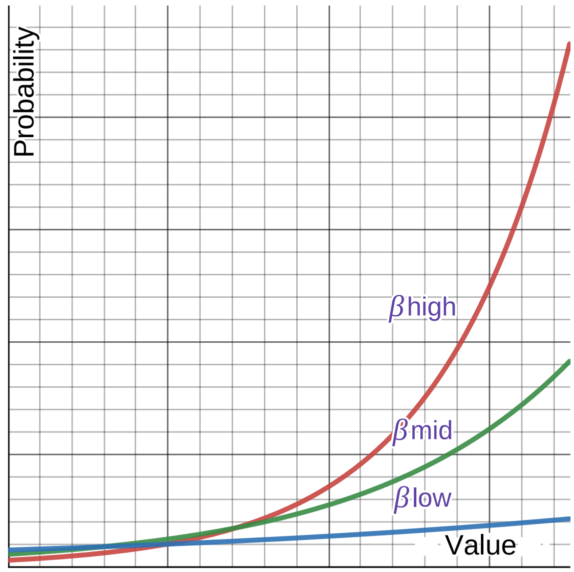

Figure 1: Probability distribution $\{p_i\}$ over V for different levels of β. Higher-benefit configurations of the parcel network are more likely than lower ones. As β increases, density shifts to the right and the left tail flattens out. This picture cuts off the leading tail of low-benefit values in $\{V_i\}$. The right-most V_i displayed is the maximal value attainable.

derived from creating clusters of neighboring parcels with the same status (like a conservation bank, or a concentration of industry), then low β implies many disjoint landowner decisions that do not consider those of their neighbors. Any social benefits to aggregation are lost as the individual focuses on their own private benefit, without recognition of any positive or negative externalities imposed on others. In the limit $\beta \to 0$, no configuration of the network is preferred. β appears to act as a sort of discount factor over space.

Conversely, a high β signifies that the various "signals" from other landowners are being received by their neighbors. If landowners can earn a return on the benefits of parcel-parcel interaction, they will naturally change their land use behavior in the direction of the regulator's preference. In the limit of "perfect fidelity" ($\beta \to \infty$), the entire network will be uniformly dedicated to the best land use.

But β will also modify the returns to any solitary action as well! If private value is created from any particular land use decision, it is attenuated by $1/\beta$. Thus it seems that $1/\beta$ is also playing the role of the opportunity cost of land ownership. As the opportunity cost increases, the potential benefit decreases toward zero (i.e. moving from high-to-low β in Figure 1). This seems to reinforce the point in the paragraph above—a high opportunity cost would decrease the perceived value of clustering neighborly land use decisions.

Now that we have developed the underlying V-distribution, the next step for us is to use Equations 7 and 8 in order to complete our relationship between the expected value Ω, policy parameters u and w, and the opportunity cost/spatial discount factor β.

2.3 Social benefits, visualized

In Section 2.2 we used the principle of maximum entropy to motivate a distribution for our social value V_i. Now we need to plug Equation 1 into Equations 7 and 8 in order to calculate the expected value Ω. But this is not so easy to solve, due to the "neighborhood sum" in our value function. Thus it is worthwhile to consider an approximate representation of this value function that simplifies the interactions between landowners.[11]

Let us first expand the land use terms in the first sum about their mean *over all potential configurations i*,

$$V_i = w \sum_{<j,k>} (\sigma_j + \delta_j)(\sigma_k + \delta_k) + u \sum_j s_j, \tag{9}$$

where $s_j = \sigma_j + \delta_j$, σ_j is the expected land use decision of site j, and δ_j represents the departure from the expected land use for site j in configuration i.

Landowner j's land use decision will naturally depend on that of their neighbors, given some of policy that incentivizes both the size and clustering of conserved areas. But we can simplify the [partially-random] decision by assuming that each landowner ultimately responds to the *expected* (anticipated) action of their neighbors, rather than waiting for their neighbor's exact choice. Indeed, someone must move first. This allows us to drop the interaction term $\delta_j \delta_k$,

$$V_i \approx w \sum_{<j,k>} \sigma_j s_k + \sigma_k s_j - \sigma_j \sigma_k + u \sum_j s_j. \tag{10}$$

(I have re-substituted the relationship between the land use decision and its mean above.)

What is a "neighbor" in this model? Landowners may consider the land use decisions of those in this parcel network that are not necessarily their physical neighbors. So when I speak of "neighbors" k, I mean those in meaningful contact with landowner j. This fairly-broad definition of neighbor has an impact on σ_j and σ_k—that they are more or less equal. How can this be? If I am responding to the expected choice of my neighbors, and each of those neighbors are responding to their neighbor's expected choices, then each landowner

[11] This type of model is attributed to Lenz and Ising, who were interested in producing a simple model of ferromagnetism (Ising, 1925). Our parcels are their atoms. Ising did not solve this problem in more than one dimension—which we want to do, as landowners have numerous neighbors, both physical and via personal connections. A century of effort has shown this to be fairly difficult beyond a simplified two-dimensional case (Onsager, 1944). The typical approach to this problem is to solve a nearly-equivalent one with non-interacting parcels which has increasing accuracy as the number of neighbors increases (Schroeder, 2000; Susskind, 2013). The error is negligible when each landowner has at least six neighbors, and still small when that number is two (Susskind, 2013).

will encounter the "signal" from a particular neighbor through multiple relationships. The only self-consistent expected land use decision, given that each landowner is fairly-well connected, is that $\sigma_j = \sigma_k = \sigma$ (Susskind, 2013). Our configuration benefit becomes

$$V_i \approx w \sum_{<j,k>} \left(\sigma(s_j + s_k) - \sigma^2\right) + u \sum_j s_j. \tag{11}$$

We can now replace the "neighbor" sum with two sequential ones, summing over each parcel, then checking the neighbors of that parcel (and adding a factor of one-half to address double-counting),

$$\begin{aligned} V_i &\approx w \cdot \frac{1}{2} \sum_j \sum_{k \in \{j\}} \left(\sigma(s_j + s_k) - \sigma^2\right) + u \sum_j s_j \\ &= w \cdot \frac{1}{2} \sum_j \sum_{k \in \{j\}} \left(2\sigma s_j - \sigma^2\right) + u \sum_j s_j \\ &= \underbrace{-\frac{1}{2} NKw\sigma^2}_{u_0} + \underbrace{(Kw\sigma + u)}_{u_{eff}} \sum_j s_j, \end{aligned} \tag{12}$$

where the second line takes advantage of the same symmetry in summing over the same parcels twice, N is the number of parcels in the network, and K signifies the number of nearest neighbors (assumed for now to be the same for all landowners). u_{eff} (effective) captures an "incentive" that captures both the conservation and clustering effects. I will drop the approximation symbol as I move forward with our new benefit function.

Thus we have now turned our coupled representation of the social value function into a decoupled one. The next step is to calculate Z, which is our door to Ω. Summing over all configurations i, we have,

$$\begin{aligned} Z &= \sum_i e^{\beta V_i} \\ &= e^{-\beta u_0} \cdot \sum_i e^{\beta u_{eff} \sum_j s_j}. \end{aligned} \tag{13}$$

If we make the substitutions $X = e^{\beta u_{eff}}$ and $Y = e^{-\beta u_{eff}}$, then Z can be simplified. Recalling that N is the number of parcels and n is the number of conserved parcels, then the configuration sum is equivalent to

$$\sum_{n=0}^{N} \binom{N}{n} X^n Y^{N-n} = (X + Y)^N, \tag{14}$$

so if we re-substitute X,

$$Z = e^{-\beta u_0} \cdot \left(e^{\beta u_{eff}} + e^{-\beta u_{eff}}\right)^N$$
$$= 2^N e^{-\beta u_0} \cdot \cosh(\beta u_{eff})^N. \tag{15}$$

We can use our relationship between Z, β, and Ω to write the expected social benefit:

$$\Omega = \frac{\partial \log Z}{\partial \beta} = N u_{eff} \tanh(\beta u_{eff}) - \frac{1}{2} N K w \sigma^2. \tag{16}$$

Using the same differentiation trick we used to find the relationship above (see Footnote 9), we can also derive a formula for the expected land-use decision, σ,

$$\sigma = \frac{1}{N\beta} \frac{\partial \log Z}{\partial u_{eff}} = \tanh(\beta u_{eff}(\sigma)) = \tanh(\beta(u + K w \sigma)). \tag{17}$$

This is an implicit function in σ that arises from our self-consistency assumption.[12] A simple change of variables will help us understand this equation more easily:

$$\frac{x}{\beta K w} = \tanh(x + \beta u), \text{ where } x = \beta K w \sigma. \tag{18}$$

We can plot both of these lines and view their intersections in order to understand the various solutions of this model. First we can consider if $u = 0$, the case where there is no additional benefit to conservation relative to economic development (the left panel of Figure 2). We will focus on varying the slope of the linear function by varying β, which I have previously related to both a spatial discount factor and the inverse of the opportunity cost of land-holding.

If β is very low (green line), the expected land use for any particular site is as good as random—exactly the prediction of the previous section. The only solution is at the origin, and no landowner takes into account the decisions of their neighbors. What if β is relatively high (blue line)? Then additional solutions appear. In the case where $u = 0$, it is clear that it does not matter *which* use dominates as long as the value can be captured by the landowner; as mentioned before, a large cluster of developed parcels or a large cluster of conserved ones both provide the same social benefit.

[12] We can use Equation 17 to simplify Equation 16 and make clear that the social value is in fact positive:

$$\Omega = N\left(u + \frac{1}{2} K w \sigma\right) \tanh(\beta(u + K w \sigma)).$$

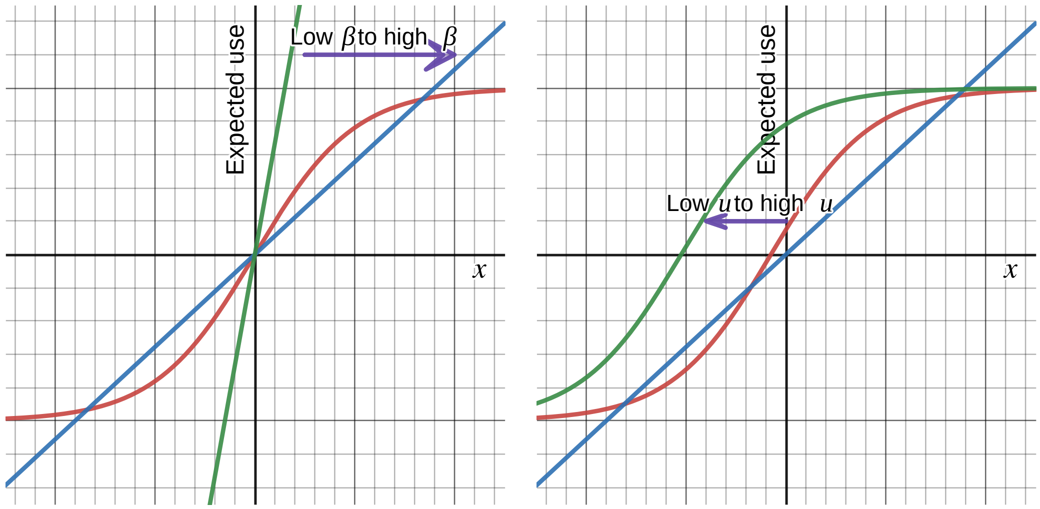

Figure 2: Left panel: When there is no biased stimulus ($u = 0$), Equation 18 yields between one and three solutions. A solution is stable only when the tanh curve is flatter than the linear curve as they intersect. For low β, the only solution is unstable. The solutions in the outer arms of the tanh curve—which appear as β increases—are stable. The network prefers one dominating land use. **Right panel**: A policy that introduces a $u > 0$ will shift the tanh curve to the left. This introduces an asymmetry: for a small u, there still remains two viable stable solutions, but with ever-stronger bias u, the weaker stable solution disappears.

The middle solution can be labeled as unstable using a heuristic argument: with just the tiniest temporary "nudge" from a momentarily non-zero u, the network would have a slight preference for aligning land use accordingly, as it increases the total value. Thus the network will move toward one of the solutions in the "arms" of the tanh curve. We can also plug in these solutions into the Hamiltonian that corresponds to the Lagrangian in Equation 4 and verify that they are in fact maxima (see Figure 3 for an illustration).

What if $u \neq 0$? A non-zero u represents a land use policy that incentivizes one use over another. The right panel of Figure 2 shows the case where $u > 0$, for both small and large u. Holding β constant, as u is increased (red-to-green tanh shift) only the highly-incentivized solution remains. When our conservation attempt is weak and u is small, we can be fairly certain that we will not accomplish much, as we have done little to change landowner incentives. While the conserved state will provide higher value, the system can still be trapped in a lower local maximum anti-aligned with the policy (see Figure 3). However, a more aggressive bias will make the returns to conservation unambiguously more attractive. In a lower-β scenario (i.e. a steeper blue line), it takes less effort to bias landowner decisions.

There are more moving parts in the $u \neq 0$ case. Both curves in the right panel of Figure 2 will shift when changing β: the linear curve flattens and the tanh curve will shift to the left with increasing β, given $u > 0$. A higher-β (i.e. a lower opportunity cost of land

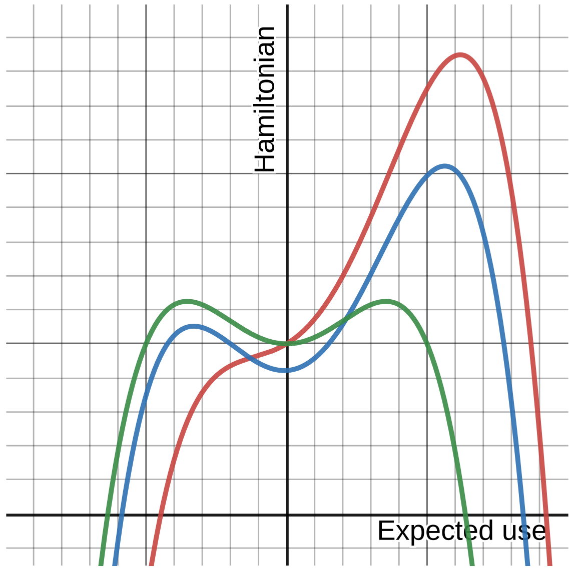

Figure 3: Illustration of the Hamiltonian as u increases (green-to-blue-to-red), holding β constant at a sufficiently-high level. When $u = 0$ (green curve), there are two stable solutions for σ. As the conservation incentive increases in strength, the Hamiltonian becomes more asymmetric, and eventually only one maximum remains for large u. For higher β, each curve would be more exaggerated. For low enough β, this shape degenerates into a downward-facing parabola symmetric about the origin, with one solution of no net use one way or another.

ownership or higher spatial discount factor) is more likely to allow for multiple solutions even with a policy that favors one land use over another. This leads us to an interesting point: if a policymaker could manipulate both β and u, they will be able to *choose* the land use solution that they want.

Figure 4 presents an illustration of the phase diagram for σ. As can be seen in Figure 2, there is a critical "β_c" below which there is no stable solution. Above this value, we have some interesting behavior. If we apply some policy such that $u > 0$, we can initially have the expected land use choice align with this policy. Given some $\beta > \beta_c$, if we then reduce $u \to 0$, as we approach the x-axis, a positive σ will remain. The magnitude will depend on β, the larger it is, the larger the "residual" σ. Close to β_c, the lasting impact is small, and above it, the impact is zero.

This explanation can be extended by returning to Figure 2. If we take the green tanh curve in the right panel and reduce u, it shifts to the right. Eventually, the second stable solution returns, but the system will remain in its local maximum of even when $u = 0$. In fact, ignoring particularly strong random variations in land use choice, the expected land use decision will not "jump" to the other maximum even for small $u < 0$. Only when the tanh curve has shifted far enough to the right will the positive solution become unstable, and the network will jump from a net-conserved to a net-developed state. So even in the

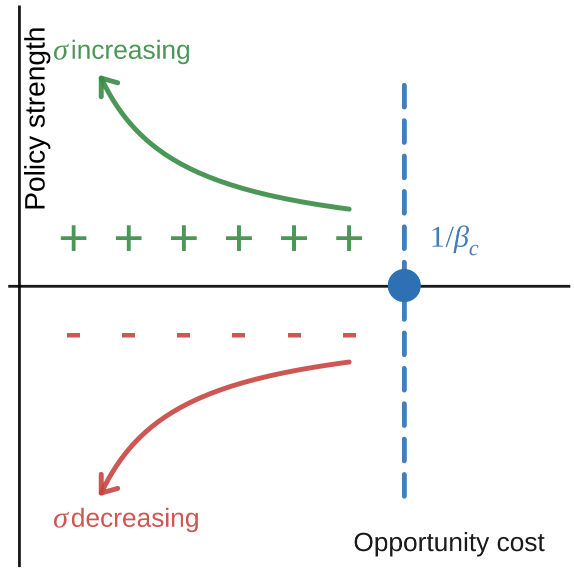

Figure 4: Phase diagram for σ, the expected land use. The axes are two of our manipulable policy variables, β and u. There is a "tear" in the surface along $y = 0$, and crossing the tear requires significant incentives. A second route for changing the state of the system is to "go around" the critical point defined by β_c.

existence of external pressures for development, our conservation incentive can persist.[13]

Our phase diagram tells us one more thing. If β decreases enough, any local equilibrium can be "broken" without requiring huge incentives for conservation ($u > 0$). For example, if the spatial discount factor is decreased, perhaps in a situation where landowner decisions are not well-communicated across the landscape, a policymaker could in theory take advantage of this situation as a low-cost time to introduce a small conservation incentive. Once in place, a second policy increasing the information passed between landowners would bias them to coordinate conservation. Put another way, if a policymaker can first increase the opportunity cost of land ownership, apply a small incentive during this period, and then decrease the opportunity cost so that we return to the left of the critical point, we can avoid requiring a large incentive to cross the x-axis more directly!

Even with these strategic manipulations, we aren't guaranteed a particular state due to the random component of each individual's land use decision. If we would like to evaluate the *variance* in the social value of the network, we can again use the differentiation trick in Equation 8 (Footnote 9) *twice*:

$$Var[V] = E[V^2] - \Omega^2 = \frac{\partial \Omega}{\partial \beta} = N \left(\frac{u_{eff}}{\cosh(\beta u_{eff})} \right)^2 . \tag{19}$$

[13]This can work against the regulator as well! Depending on the current state of the world, some incentives will result in wasted money and effort as they won't be able to escape an undesirable local equilibrium.

Equation 19 provides us with another metric with which we can judge the desirability of a given policy. For example, in the case where we want to depart slightly from the maximal expected value in favor of avoiding a large variance in outcomes, we can satisfy the needs of a risk-averse regulator.

We now have a way to determine Ω, and we have seen how a regulator can manipulate u and β in order to choose the expected state of the system from the set of viable solutions. The next section introduces some useful model extensions which will motivate the optimal control framework in the next iteration of this paper.

2.4 A few valuable model extensions

The social benefit function in Section 2.1—Equation 1—values both clustering of parcels of a particular use, and the number of parcels used in conservation. One simplification is that "developed" parcels have no intrinsic absolute value, but this can be addressed with an additional term:

$$V_i = w \sum_{<j,k>} s_j s_k + u \sum_j s_j + v \sum_j s_j^2. \tag{20}$$

This small change makes the value of an additional developed parcel $v - u$, before adding in the neighbor interaction effects. The value of an additional conserved parcel is $v + u$, thus the effective "size" of the conservation incentive is $2u$.

After following the same steps used to derive Equation 12, the resulting V_i changes to

$$V_i \approx \underbrace{-\left(\frac{1}{2}Kw + v\right) N\sigma^2}_{u_0} + \underbrace{((Kw + 2v)\sigma + u)}_{u_{eff}} \sum_j s_j, \tag{21}$$

and with the new definitions of u_0 and u_{eff} above, everything else from Section 2.1 follows.

We can also some opposing pressure to the clustering effect. In a case where developed parcels produce a good or service sold in a market, as fewer parcels are developed, the value of developing a parcel may increase due to product demand. We can add another "neighbors" sum, where value for a given parcel is established by anti-aligning with other parcels of comparable potential or the number of dissimilarly-used parcels in the vicinity:

$$V_i = w \sum_{<j,k>_1} s_j s_k + u \sum_j s_j + v \sum_j s_j^2 - b \sum_{<j,k>_2} s_j s_k - c \sum_{<j,k>_3} s_k. \tag{22}$$

In this case, the neighbor sets may be different (labeled with a "1," "2," and "3" here).

Since the terms are additive, the steps from the previous section can be used to define the appropriate v and u_{eff}.

If we wish to introduce discrete values of s in the interval $[-1, 1]$, the only step in Section 2.1 that is ultimately affected is the binomial theorem trick used to derive Equation 15. If land use gradations are developed, i.e. $s = \pm 1/2$, we can use the multinomial theorem to develop additional additive cosh terms in Z. And if we add unbalanced steps—i.e. $s = 1/2$ but not $s = -1/2$—these changes would add an exponential term that shifts the tanh curve left or right. The curve retains the sigmoid shape, however the analytic tractability of the solution decreases.

We could assume the land use decision is in fact a vector, i.e. $s = (1, 1, -1, 1)$. In this case, we can generalize the $s_j s_k$ terms with an inner product, and the remaining derivation is unchanged. Other than visualization becoming more difficult, this may have a difficult-to-predict effect on the solutions for the now-vectorized σ.

Lastly, we can introduce new terms in V_i with sums that operate over a particular landowner or parcel characteristic. We have already discussed this in the context of different neighbor sets. Due to separability of the terms, this can also be managed.

In the next iteration of this paper, I will include terms that allow development to have its own absolute value (term "v") and conservation in urban areas to be preferentially-rewarded due to local scarcity (term "c"). These two additional terms capture the full nuance of the policy described in Section 1. For now, I will conclude with a summary of the theoretical contributions already developed.

3 Discussion

As land is developed, habitat fragmentation weakens natural ecosystem function and decreases the probability of long-run species viability. Combating this problem is difficult because land ownership itself is often fragmented. Not only do the social benefits accrue to those other than the landowners, the direct conservation value of any one isolated parcel is quite small—yet these social benefits in large part depend on the clustering and spatial distribution of many of these small pieces of habitat. What is best for the landowner will doubly erode social values without a creative compensation scheme. This particular collective action problem presents a unique regulatory challenge.

Existing policies that combat habitat loss—like conservation banking—are incomplete because they fail to reflect the positive network effects that the clustering of many small conserved parcels create as well as the spatial distribution of conservation benefits (which determine who receives the social benefit). To address this shortfall, this paper adapts the banking approach to regulate a decentralized group of landowners and uses this guidance to put forward an incentive scheme that can motivate collaboration across many independent smallholders. The model in Section 2 shows that any desired equilibrium can be achieved by carefully tuning the compensation mechanism and determines how to avoid wasted effort.

The next step in the evolution of this paper is the inclusion of an optimal control model that captures the regulator's task of changing the composition of their landscape from some starting state to a desired configuration, given some budget constraint. Doing this will complete the example theoretical implementation of the smallholder-friendly conservation banking approach. As the regulator only plans the expected path of improvement, an important part of this exercise is to use simulation to determine what can be tweaked to ensure robustness in outcomes.

This paper relies on concepts from statistical mechanics to model a complicated network of interacting landowners, predict their reactions to different land-use policies, and measure the social conservation benefits of the aggregation. Such a modeling paradigm is valuable to many sub-disciplines of economics beyond land conservation and natural resource management. Three features stand out. First, the maximum entropy principle in Section 2.2 provides a way to model the aggregated consequences of many individuals' land use decisions without the need to make any unconscious or arbitrary assumptions. This approach can replace the arbitrary assumptions and artificial certainty imposed by "extreme-value" or "normally-distributed" errors that are often made for tractability reasons in discrete-choice modeling.

Second, the maximum entropy approach fosters a broader understanding of constrained maximization problems. The "solution" has little to do with the particular value some choice variable takes on and much more to do with the functional connections between the [arbitrarily-designated] inputs and output variables. The process of inverting this resulting functional relationship in order to "free" Ω and give physical meaning to β, for example, introduces a new tool for deriving *inspired* functional forms for quantities like our present social benefits function. This tool could play a role in replacing the more egregious ad-hoc displays in economics.

Third, the Ising model in Section 2.1 and 2.3 suggests a new way to think about strategic behavior among many independent actors. At present, this paper is the only one in economics to adapt the model from the physical sciences, so there is lots of room for exploration. This is not to say such an endeavor should be pursued only because it is novel or exotic, but the simple discovery of the model's applicability here is such a valuable one that it leads one to think there is more out there worth borrowing.

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